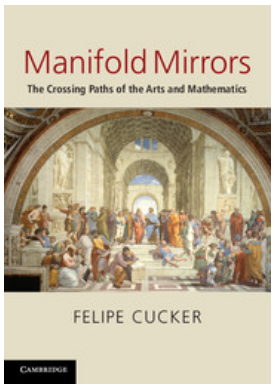
Manifold Mirrors. The Crossing Paths of the Arts and Mathematics, 2013, Cambridge University Press, ISBN 978-0-521-42963-4 (hbk), x+415 pp. by *Felipe Cucker*.



Felipe Cucker

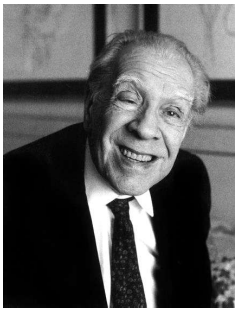
The cover states that this book grew out of a liberal arts course. Felipe Cucker is Chair Professor of Mathematics at the City University of Hong Kong. The result is a collection of chapters, some of which are just plain mathematics, others analyse the philosophical and psychological aspects of aesthetics, and of course many discuss a wide diversity of works of art and how the mathematics are recognized in their features (like symmetry or translation invariance for example) or how mathematics have influenced the techniques available to the artists (like perspective and hyperbolic geometry).

Almost all examples are two-dimensional, which is related to the mathematics that are covered. These are all geometric, many treat transformations in the plane, but others deal with projections, and non-Euclidean geometry. So most art examples are graphical like paintings and carpets, but occasionally also poetry and dance performances. The chapter devoted to Bach's canons, is and exception. Architecture and sculpture, which are three-dimensional, are almost absent.

But let us go more systematically through the contents of the book. It starts with some appetizers: The famous *Maestà* fresco of Simone Martini (1284-1344), John Milton's (1608-1674) *Paradise lost* (his blindness draws a parallel with Jorge Luis Borges (1899-1986)), and Johan Sebastian Bach's (1685-1750) *Musical Offering*. It gives a short biography of the artists, but mainly discussing the symmetry, rhythm, structure in their work.

S. Martini

J. Milton



J.L. Borges

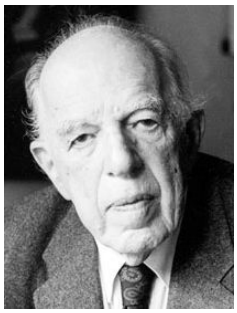
J.S. Bach

Then a chapter introduces geometry and its history from Euclid to Descartes and this is followed by a mathematical treatment of plane transforms: translation, rotation, reflection, glide, isometry, completely with definitions and proofs. Artistic examples illustrate the mathematics in another chapter where it is shown that there are exactly 7 friezes (translation invariant pattern in one direction) and 17 wallpapers (translation invariant pattern for two independent vectors).

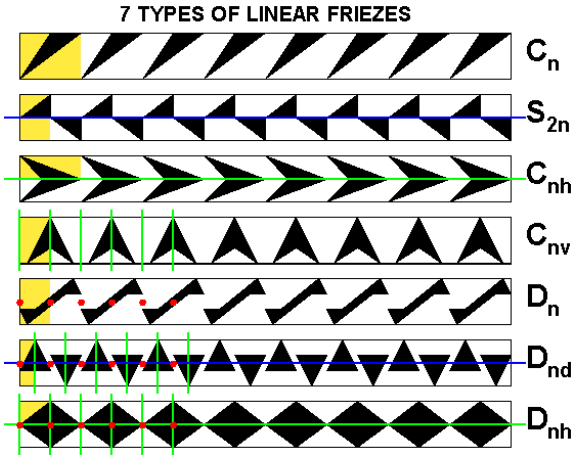
Pieces of art with planar symmetry are easily found. Tessellations of the plane are very common in islamic decoration. However Cucker chooses rugs and carpets from Central Asia and Chinese lattices as the main illustrations to be accompanied with the obvious choice of M.C. Escher's (1898-1972) work.

Much more philosophical is an analysis of George D. Birkhoff's (1884-1944) *Aesthetic Measure* who actually derived a formula which was the ratio of the amount of symmetry over the complexity of the object. It is followed by

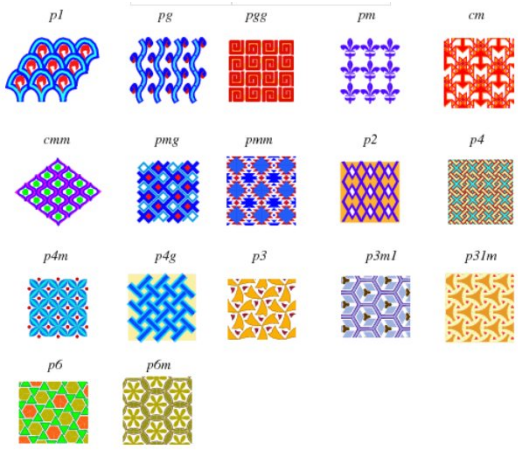
G.D. Birkhoff



E.H. Gombrich

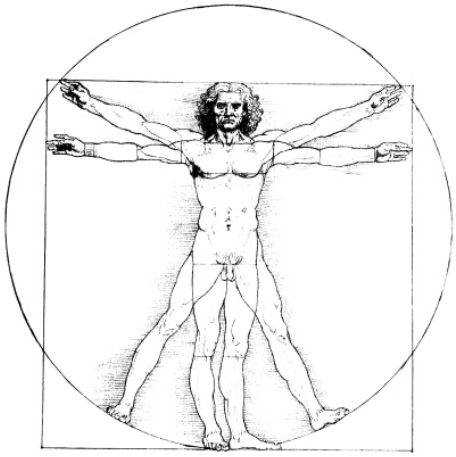


7 friezes



17 wallpapers

the theory of the art historian Ernst H. Gombrich (1901-2001) about our *sense of order*. An object raising a sense of beauty has found a balance between complete disorder and boredom. Think of a regular square grid. That might be interesting if you want to solve your differential equation, but it is aesthetically very dull. On the other hand our brain perceives symmetry when it is not really there. For example the *Vitruvian man* by Leonardo Da Vinci is not symmetric. The left leg and foot is rotated with respect to the right one. This delicate balance is illustrated by several examples from op-art, by the repetitions in work of Andy Warhol (1968-1987) with his *Campbell's Soup* cans or his screenprints of *Marilyn Monroe*, but also with several ballet scenes like for example the rotational symmetry evocated by the *Pilobolus Dance Theater* which is not perfect either. Also the regularity in rhyme and rhythm like iamb, trochee, dactyl, and anapest.

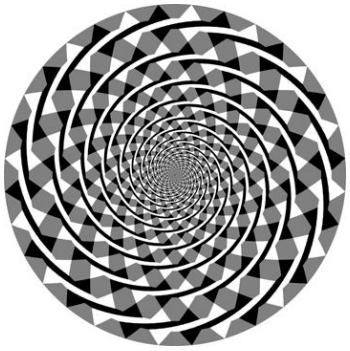


Vitruvian man

The next chapter starts out again with mathematics introducing homothecies, the group of similarities, shears, strains and affinities. This brings along conics and the optical spiralling effect of the *Fraser spiral* caused by the transformed black rectangles even though there are only concentric circles. The ellipse used during the Renaissance gets some special attention. It can be seen as a prolonged version of the perfectly symmetric circle. These can be observed for example in paintings by Peter Paul Rubens (1577-1640), but also in the elongated bodies in the paintings by El Greco (1541-1614). The chapter ends with Klein's *Erlangen Program* to classify geometry (not fully explained here) and other possible representations of reality like topologies (think of a map of a metro system).

Marilyn Monroe
A. Warhol

Pilobolus
Dance Theater



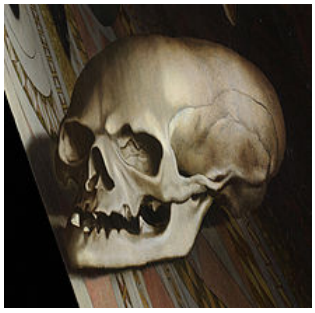
Fraser spiral

Saint Martin
El Greco

Exchange of Princess
P.P. Rubens

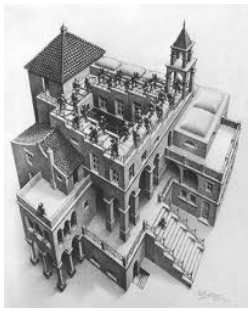
The chapter with a slightly unexpected title *Aural wallpaper* introduces several terms related to music scores and then defines the 'geometry of canons' (in terms of translations and symmetries). All in preparation of a more extensive discussion of the 10 canons from J.S. Bach's *Musical Offering* and motifs used by others, arriving at work by Satie and Riley and minimal music. The end of the chapter explores an essay by Peter Kivy (1993) who is defending the role of music as 'aural wallpaper'.

The Ambassadors
H. Holbein



rescaled skull
left fig. center front

reflection in sphere
M.C. Escher



ascending and descending
M.C. Escher

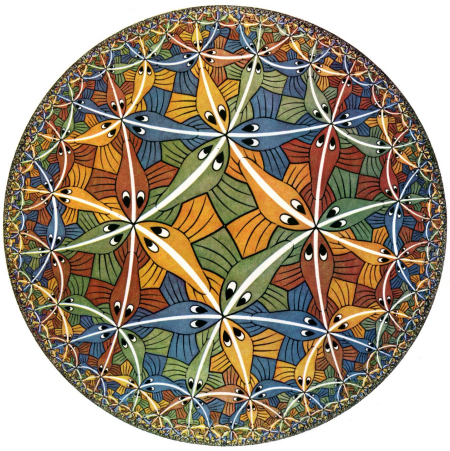
Back to geometry, Cucker shows how perspective was gradually introduced by European painters and gives some of the mathematics and elements from projective geometry and different kinds of projections. This lead to a conflict between composition and the rules of perspective, or distortions like the paintings on the ceiling of some churches. A remarkable example of distortion is the skull on the painting *The Ambassadors* by Hans Holbein (1497-1453). It appears as a strange elongated object at the center front and it can only be recognized when the painting is looked at from a very small angle. There are of course the many reflections in spheres and convex mirrors made possible, but also the many impossible constructions obtained by false perspective (e.g. M.C. Escher). Less known is the divergent perspective like in the *Trinity* icon by Andrei Rublev (ca. 1360-1428). The feet of the angels rest on trapezoidal forms whose sides converge towards the observer. The chapter ends where painters abandon perspective like e.g. in *Les Demoiselles d'Avignon* (1907) by Pablo Picasso (1881-1073), considered to be the start of cubism and modern art.

Trinity (A. Rublev)

Just as in art the classical rules were abandoned, also in mathematics the geometry of Euclid was abandoned. Cucker goes through the trouble of defining formal axiomatic systems and formal languages, and even formulates Gödel's completeness theorem. This allows him to define models for Euclidean geometry, but also Poincaré's model for hyperbolic geometry, and non-Euclidean geometries like projective and spherical geometry. This opens the route towards tessellations of the sphere, and the projective and hyperbolic plane where again Escher is the obvious illustrator of these.

In a final short chapter, Cucker ponders on the question whether there is a 'true' geometry. One that represents reality. So he makes an excursion, leaving 2-dimensional geometry and arrives at general relativity, space-time structure and the topology of our universe.

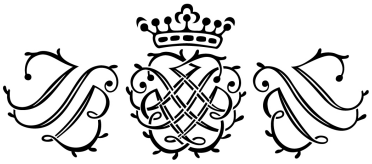
In an appendix some ideas about 'rule-driven creation' is given. Is the creation of art is not just a matter of following the rules, because then it could be automated, but neither is it not following any rule. So far, with only few exceptions, what has been discussed was mainly 2-dimensional visual art (apart from the chapter on audio wallpaper). Cucker takes up this occasion to discuss literature, which also follows certain rules, except that he did not clearly see the geometric concepts in this creation process. There are however examples of constrained writing like the extreme use of anagrams, palindromes and other word plays. Famous is *La Disparition* (1963) by Georges Perec (1936-1982), a lipogram novel in which the letter 'e' does not occur or his *Les Revenences* (1972) in which 'e' was the only vowel.



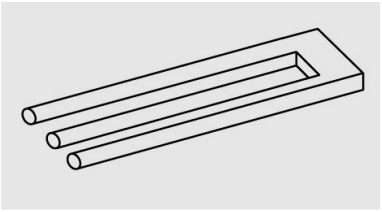
Circle Limit III (Escher)

Satire of false perspective
W. Hogarth

Les demoiselles d'Avignon
P. Picasso



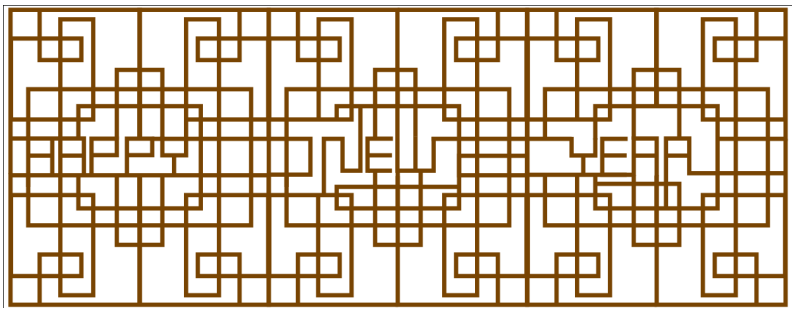
Symmetry in J.S.Bach's seal



Impossible object

This survey shows that even though the author has restricted his approach mainly to two-dimensional geometry and transformations of the plane, it should be clear by now that this is still a very broad area when this is related to visual art. The result is an unusual mixture of proper mathematics, practical illustrations from global cultural heritage covering human history from the stone age till modern art, and a discussion of philosophical and psychological essays. The choice of the art works are not always the most well known. So there will be most likely something new to discover for every reader. The book grew out of a course, and so it is certainly possible to extract some interesting lectures from the material that is presented.

Adhemar Bultheel



Chinese lattice